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Education and Self-Employment: Changes in Earnings and Wealth Inequality

by

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The views expressed in this paper are those of the author.
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Abstract

The author quantitatively studies the interaction between education and occupation choices and its implication for the relationship between the changes in earnings inequality and the changes in wealth inequality in the United States over the 1983–2001 period. Among households whose head is a college graduate, the ratio of average household earnings between the self-employed and workers increased by 57 per cent. At the same time, the ratio of the average household wealth increased by 137 per cent. These findings suggest that both earnings and wealth inequality increased over this period. Did this change in relative average earnings lead to the change in relative average wealth? The author builds on a model of wealth distribution to include education and occupation choices, where earnings opportunities are dictated by productivity processes that are education-occupation specific. By calibrating these productivity processes to match the earnings observations separately for 1983 and 2001, the author quantitatively derives the model-implied changes in wealth inequality between different education-occupation groups of households. The results show that this exercise leads to one-third of the change in the relative average wealth between college self-employed and college worker households.

JEL classification: D31, I21, J23
Bank classification: Economic models; Labour markets

Résumé

L'auteur étudie l'interaction entre les choix de formation et les choix de carrière et tente de cerner, sous l'angle quantitatif, son incidence sur la relation observée entre l'évolution des inégalités de revenu et celle des inégalités de richesse aux États-Unis de 1983 à 2001. Dans les ménages dont le chef possède un diplôme universitaire, le ratio du revenu moyen des travailleurs autonomes à celui des salariés s'est accru de 57 %, alors qu'au cours de la même période, le ratio correspondant pour la richesse moyenne a fait un bond de 137 %. D'après ces chiffres, les écarts de revenu comme de richesse se seraient creusés durant la période examinée. On peut se demander si cette évolution du revenu moyen relatif est à l'origine de celle de la richesse moyenne relative. Pour répondre à la question, l'auteur intègre des choix de formation et de carrière à un modèle de répartition de la richesse, afin de faire dépendre les perspectives de revenus de processus de

productivité propres aux choix effectués. Après avoir calibré séparément ces processus en fonction des données de 1983 et de 2001 sur le revenu, l'auteur calcule à l'aide de son modèle les variations d'écart de richesse entre des classes de ménages différenciés par leurs choix de formation et de carrière. Les résultats obtenus lui permettent d'expliquer le tiers de la variation de l'écart observé en ce qui concerne la richesse moyenne des ménages formés de travailleurs autonomes et de salariés qui ont un diplôme universitaire.

Classification JEL : D31, I21, J23
Classification de la Banque : Modèles économiques; Marchés du travail

1 Introduction

Earnings and wealth inequality have significantly increased along many dimensions over the past several decades in the United States. The increase in the college premium has been widely documented in the literature. For example, Katz and Murphy (1992) and Bound and Johnson (1992) show the increasing trend in the relative wage of more educated workers. Díaz-Giménez, Quadrini, and Ríos-Rull (1997) and Budría-Rodríguez et al. (2002) document the changes over time in average earnings and wealth along educational and occupational dimensions.

In this paper, I empirically study the interaction between the educational and occupational choices important in analyzing between-group earnings and changes in wealth inequality over time. Based on these empirical findings, I quantitatively study the extent to which the changes in earnings inequality account for the changes in wealth inequality between education-occupation groups.

In conducting these studies,

- I document that the relative number of college-educated self-employed households increased over the 1983–2001 period in the United States;
- I document that between-group earnings and wealth inequality increased significantly between college self-employed households and others;
- I build a general-equilibrium model where agents face uninsured idiosyncratic shocks to their productivity and make education, occupation, and consumption-savings decisions so that the resulting earnings, as well as wealth, are endogenously derived based on these decisions; and
- I measure how much of the observed changes in between-group wealth inequality can be accounted for within the model by the observed changes in earnings inequality.

First, in the empirical part of this paper, I show that the interaction between education and occupation decisions is an important determinant of between-group wealth inequality. I also show, using the Survey of Consumer Finances (SCF) data set, that the fraction of households identified as college self-employed increased from 4 per cent to 7 per cent over the 1983 to 2001 period. At the same time, earnings and wealth inequality increased significantly between college self-employed households and others. Among college graduates, the ratio of average household earnings between

self-employed and workers increased by 57 per cent, while the ratio between college graduates and non-college graduates among the self-employed increased by 27 per cent. Wealth inequality among different groups of households increased much more dramatically. Among college graduates, the ratio of average household wealth between the self-employed and workers increased by 137 per cent, while the ratio between college graduates and non-college graduates among the self-employed increased by 126 per cent. These observations show that between-group wealth inequality, as well as earnings inequality, increased over this period, and that the interaction between education and occupation may play an important role in those inequalities, as demonstrated by the increase in the fraction of college self-employed households.

Second, the quantitative exercise based on the model that captures the observed changes in between-group earnings inequality shows a mixed result. An incomplete market model with education and occupation choices is used to measure the effect of changes in between-group earnings inequality on changes in between-group wealth inequality. This quantitative analysis involves directly calibrating changes in the productivity processes (i.e., labour-efficiency units for workers and managerial productivity for the self-employed) to capture the observed changes in between-group earnings inequality. Average earnings in the model are endogenous, because they depend on the number of households that choose to be in each education-occupation group. Calibrating the productivity processes within the model is therefore necessary to account for this self-selection into a particular group. The results of the analysis show that the changes in earnings inequality can account for one-third of the change in wealth inequality between college self-employed and college worker households, while it accounts for almost none of the change between college self-employed and non-college self-employed households.

The intuition behind this quantitative result is as follows. While the increase in between-group average earnings inequality implies an increase in between-group wealth inequality, there is an offsetting, much stronger, second-moment effect from the changes in the variance of earnings on the change in wealth inequality. During the period, the variance of earnings for non-college self-employed households went up much more than for college self-employed households. Since the agents in the model are risk-averse, an increase in the variance of earnings leads to an increase in precautionary savings. The calibration results show that, between college self-employed and non-college self-employed households, this precautionary effect cancels out the direct effect such that the overall change in

the wealth inequality is very small. A new theory of the change in between-group wealth inequality is needed to capture the change in wealth inequality, especially between college self-employed and non-college self-employed households. This study identifies some institutional changes that could account for the changes in wealth inequality observed in the data over the period.

There is a large body of empirical literature that documents inequality and the change therein. Katz and Murphy (1992), Bound and Johnson (1992), and Gottschalk and Moffitt (1994) report that the overall earnings distribution widened from the 1970s to the 1980s, but that different educational groups faced different degrees of increase in the earnings variances. Díaz-Giménez, Quadrini, and Ríos-Rull (1997) and Budría-Rodríguez et al. (2002) use 1992 and 1998 SCF, respectively, to document the earnings and wealth inequality observations along educational and occupational dimensions; however, they examine neither the interaction between these two dimensions nor the changes in inequality within this interaction. Gentry and Hubbard (2000), using 1983 and 1989 SCF, find, among other things, that entrepreneurial households are "richer," on average, in many dimensions.

In addition, a quantitative analysis literature started by Aiyagari (1994) and Huggett (1993) uses the incomplete market framework to analyze the distribution of wealth. Castañeda, Díaz-Giménez, and Ríos-Rull (2003) review results from several papers; they also show that their model with uninsured idiosyncratic earnings risk, retirement, altruism, and government transfers can account for the earnings and wealth inequality observed in the United States, including the tails of the distributions. Their calibration exercise shows that, to capture their inequality, a small fraction of earners who have a very high labour income is needed. Quadrini (2000) explicitly models these earners of high labour income by including the occupational choice to become an entrepreneur. He examines how the opportunity to become an entrepreneur affects the wealth distribution in a general-equilibrium environment.[1] In this paper, I follow this line of research to analyze between-group earnings and wealth inequality, where household groups are defined by education and occupation.[2]

The remainder of this paper is organized as follows. Section 2 describes the empirical findings. Section 3 introduces a general-equilibrium model of the wealth distribution with education and

[1] Cagetti and De Nardi (2002) study an occupational choice model similar to Quadrini's (2000), but the borrowing constraints of individuals are endogenously derived.

[2] A similar literature that looks at the changes in income inequality and consumption inequality studies a similar issue as this paper. For example, Krueger and Perri (2005) argue that the increase in income inequality was not accompanied by a corresponding rise in consumption inequality because financial markets developed to provide better insurance for individuals as the volatility of income rose.

occupation choices. Section 4 describes the calibration method used and section 5 discusses the results of the quantitative analysis. Section 6 suggests some extensions of the model for future work. Section 7 offers some conclusions.

2 Empirical Facts

In this section, I document the empirical findings on household type distribution, earnings, wealth, and the changes they experienced over the 1983–2001 period.

2.1 Composition of household groups by education and occupation

2.1.1 *Changes in the composition of household groups by education and occupation*

Over the 1983–2001 period, the increase in earnings and wealth inequality was driven mainly by a specific group of households whose head was college-educated and self-employed. In this section, I document the changes in the composition of four household groups defined by two educational levels and two occupations. The education and occupation of the household is defined by those of its head. The two educational levels are college and non-college, and the two occupations are self-employment and non-self-employment, the latter of which is called "worker" for simplicity. Unless otherwise noted, all statistics are from the SCF.[3] The description of this data set, as well as the sample selection, is given in Appendix A, and the various definitions used are given in Appendix B.

The distribution of households over the four groups considered changed over this period. Tables 1 and 2 show this change. The fraction of non-college workers declined from 0.61 in 1983 to 0.56 in 2001. Offsetting this, the fraction of college households increased from 0.26 to 0.28 and 0.04 to 0.07 over this period for college worker and college self-employed groups, respectively. The fraction of non-college self-employed households did not change, staying at 0.09.

In studying the interaction between the educational and occupational choices, it is useful to analyze the self-employment rate, which is defined as the fraction of the self-employed over all working households. Tables 1 and 2 also show the change in the self-employment rates for college and non-college households over the period. There was a dramatic increase in the self-employment rate for college households, from 13 per cent in 1983 to 20 per cent in 2001, whereas the rate remained relatively more stable at 13 per cent to 14 per cent for non-college households. Thus, college

[3] Budría-Rodríguez et al. (2002), and several *Federal Reserve Bulletin* articles, (e.g., Kennickell and Starr-McCluer 1994; Aizcorbe, Kennickell, and Moore 2003), provide a statistical summary of the SCF.

Table 1: Household Type Distribution, 1983

Edu/occ	Worker	Self-employed	Self-employed (%)
Non-college	0.61	0.09	13
College	0.26	0.04	13

Table 2: Household Type Distribution, 2001

Edu/occ	Worker	Self-employed	Self-employed (%)
Non-college	0.56	0.09	14
College	0.28	0.07	20

households became more likely to be self-employed than non-college households. These changes in the distribution of household type are an endogenous outcome of the household decisions on education and occupation. These decisions are affected by the earnings prospect of each education-occupation type. Hence, it is important to combine these observations with those of earnings as well as wealth.

2.2 Changes in U.S. earnings inequality by education and occupation

Changes in the earnings of the four education and occupation groups are documented in this section. Appendix B provides the definition of earnings.

2.2.1 *Between-group inequality*

Over the 1983–2001 period, the changes in the relative average earnings among the four groups were significant. Tables 3 and 4 show the relative earnings of the four groups in 1983 and 2001, respectively, where the earnings of the college self-employed household are normalized to one in each table. The last row of Tables 3 and 4 calculates the college-earnings premium as the ratio of the college average earnings to the non-college average earnings given an occupation. Similarly, the last column calculates the self-employment earnings premium over the worker earnings given an educational level. In 1983, the college-earnings premium was highest for the self-employed household at 2.07, compared with 1.66 for workers. The self-employment earnings premium was also higher for college households at 1.16, compared with 0.93 for non-college households. This pattern continued in 2001, but at a higher premium. The college-earning premium was 2.63 and 2.09 for self-employment and worker households, respectively, whereas the self-employment earnings premium was 1.82 and

Table 3: Relative Average Earnings, 1983

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.52	0.46	0.93
College	0.86	1.00	1.16
College/Non-college	1.66	2.07	

Table 4: Relative Average Earnings, 2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.26	0.38	1.44
College	0.55	1.00	1.82
College/Non-college	2.09	2.63	

Table 5: Percentage Change in Average Earnings, 1983–2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	13	76	56
College	43	124	57
College/Non-college	26	27	

1.44 for college and non-college households, respectively.

In Table 5, the four numbers in the middle cell show the percentage change in the real average earnings of the four groups between 1983 and 2001. This table shows how the results in Tables 3 and 4 came about. Over this period, the college self-employed households increased their earnings by 124 per cent, on average, whereas the non-college self-employed and the college worker households increased their average wealth by 76 per cent and 43 per cent, respectively. Comparatively, the non-college worker households increased their average earnings by only 13 per cent. The last row of Table 5 shows the percentage increase in the college-earnings premium over non-college: 26 per cent for the worker household and 27 per cent for the self-employed household.[1] The last column of Table 5 shows the percentage increase in the self-employment earnings premium over workers: it rose by 56 per cent and 57 per cent for non-college and college households, respectively.

[1]These numbers are in line with the findings of Krusell et al. (2000), who calculate that the college premium in wages per hour rose by about 20 per cent during the 80s.

Table 6: Relative Coefficient of Variation in Earnings, 1983

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.66	0.75	1.13
College	0.91	1.00	1.10
College/Non-college	1.39	1.34	

Table 7: Relative Coefficient of Variation in Earnings, 2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.36	1.36	3.81
College	0.91	1.00	1.10
College/Non-college	2.54	0.73	

Table 8: Percentage Change in Coefficient of Variation, 1983–2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	7	260	236
College	96	96	0
College/Non-college	84	-45	

2.2.2 *Within-group variability*

In terms of the within-group variability of earnings, Tables 6 and 7 show the relative coefficient of variation in earnings, in 1983 and 2001, respectively, where the coefficient for college self-employed households is again normalized to one. The last row and column of the tables show the ratios of coefficients of variation in terms of college over non-college households conditional on occupation, and self-employed over worker households conditional on education, respectively. I observe two major changes in the relative coefficient of variation over the period. First, the non-college worker's coefficient went down relative to college self-employed, from 0.66 in 1983 to 0.36 in 2001. Second, the non-college self-employed's coefficient went up, from 0.75 to 1.36. These changes are driving the observed differences in the ratios of coefficients in the last row and the last column of Tables 6 and 7. Specifically, the ratio of coefficients, college over non-college for the self-employed, went down from 1.34 to 0.73 over the period, implying that the self-employment earnings risk went up for the non-college household relatively more than for the college household. A similar big change occurred in the ratio between the non-college self-employed household and the non-college worker household:

Table 9: Mean and Median Net Worth Ratios of 2001 Value over 1992 Value, by Education

Education	No HS diploma	HS diploma	Some college	College degree
Mean	1.19	1.31	1.35	1.89
Median	1.10	1.22	1.00	1.76

Note: HS stands for high school.

the relative coefficient went up from 1.13 in 1983 to 3.81 in 2001.

Table 8 shows the percentage change in the earnings coefficient of variation for each of the four groups over the period. All the within-group variabilities went up over the period. Since the education, occupation, and consumption-savings decisions very likely depended not only on the average earnings but also on the variability of the earnings process, these changes in variability are important factors to be considered in accounting for the increase in wealth inequality. The coefficient of variation for the non-college self-employed household went up the most by 260 per cent, compared with 96 per cent for both college groups and 7 per cent for the non-college worker.

2.3 Changes in U.S. wealth inequality

Wolff (1996) and Budría-Rodríguez et al. (2002) report that the change in overall wealth inequality during the past two decades is not big. However, their studies focus on overall wealth inequality and not on between-group inequality. As I document in this section, between-group wealth inequality has significantly increased between the college self-employed household and others. One reason why I focus on two educational groups, college graduates and non-college, is that the data suggest a clear distinction in the savings behaviour between them. The SCF data show that the relative change in wealth over time is significantly different between these two groups, but less so within the subgroups of the non-college graduate households. Aizcorbe, Kennickell, and Moore (2003) summarize several changes in the U.S. household finances using 1992–2001 SCF. Table 9 shows the ratios of net worth in 2001 over that of 1992 for different educational groups. These numbers are calculated using Table 3 in Aizcorbe, Kennickell, and Moore (2003). As is clear from Table 9, there is a big jump in these ratios between the categories of Some college and College degree, which supports the claim that the increasing between-group wealth inequality is most significant between the college and the non-college graduate households if the education dimension is focused upon. I therefore focus on this dimension in analyzing educational choices.

Table 10: Relative Average Wealth, 1983

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.27	0.61	2.26
College	0.52	1.00	1.92
College/Non-college	1.93	1.64	

Table 11: Relative Average Wealth, 2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	0.06	0.27	4.50
College	0.22	1.00	4.55
College/Non-college	3.67	3.70	

Table 12: Percentage Change in Average Wealth, 1983–2001

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	-43	8	99
College	4	146	137
College/Non-college	90	126	

2.3.1 *Changes in wealth inequality by educational and occupational groups*

Over the 1983–2001 period, changes in the relative average wealth holdings among the four groups are striking. While this trend is similar for earnings, it is much stronger for the wealth observations. Tables 10 and 11 show the relative average wealth of the four groups in 1983 and 2001, respectively, where the wealth of the college self-employed households is normalized to one. Over this period, there was an increasing concentration of wealth to the college self-employed households. The wealth of all three groups relative to that of college self-employed declined from 0.27 to 0.06 for the non-college worker, from 0.61 to 0.27 for the non-college self-employed, and from 0.52 to 0.22 for the college worker. Table 12 shows how this dramatic change in relative average wealth came about between 1983 and 2001. The college self-employed households increased their wealth, on average, by 146 per cent over the period, whereas the non-college self-employed and college worker households increased their average wealth by merely 8 per cent and 4 per cent, respectively. The non-college worker household, on the other hand, experienced a 43 per cent decrease in their average wealth from 1983 to 2001. Most importantly, the relative average wealth for the college self-employed household

increased by 126 per cent with respect to the non-college self-employed, while it increased by 137 per cent with respect to the college worker.

Combining all the empirical facts thus far, two observations can be made. First, since the average wealth of the college self-employed household rose as well as the fraction of this household, the rise in its average wealth is not a result of the wealth-poor college self-employed switching to become a college worker. Second, a comparison of Tables 5 and 12 shows that while the average earnings of all four groups increased, only the college self-employed's average wealth went up significantly. These observations reinforce the importance of analyzing the interaction of education and occupation choices to study the changes in between-group inequality. Section 3 provides a model to determine the extent to which the changes in earnings inequality can explain the changes in wealth inequality between education-occupation groups.

3 Model

In introducing a model, two important issues must be considered. First, the liquidity constraint in the investment in small businesses is an important part of the quantitative theory; see, for example, Quadrini (2000) and Cagetti and De Nardi (2002). A large empirical literature supports this view: e.g., Evans and Jovanovic (1989); Holtz-Eakin, Joulfaia, and Rosen (1994); Quadrini (1999); and Gentry and Hubbard (2000).[5] I follow this view and assume that the liquidity constraint is an important determinant of self-employment.

Second, households choose their educational level and occupation based on the relative earnings prospect of each type. Hence, in order to account for the change in wealth inequality based on the change in the earnings prospect of the different educational and occupational types, the fact that households self-select into an education-occupation group has to be taken into account. A general-equilibrium model is therefore required that captures the endogenous change in the distribution of household types in accounting for the change in wealth inequality based on the change in the earnings prospect; i.e., a labour-productivity process is required.[6]

[5] There is some evidence against this view. Hurst and Lusardi (2004) argue that, when household wealth is instrumented, the liquidity constraint binds only for the very wealthy household that starts a business.

[6] In this paper, the underlying type-specific productivity processes are taken exogenously. One theory in accounting for the changes in these processes themselves is to consider capital-skill complementarity in each occupation. Krusell et al. (2000) argue that most of the change in the skill premium in the United States since the 1960s can be accounted for by the change in the relative factor supplies of unskilled labour, skilled labour, and two types of physical capital. They assume that the aggregate production technology distinguishes the elasticity of substitution between unskilled labour

In this section, I introduce a general-equilibrium model with the education, occupation, and consumption-saving choices, where the fundamental driving forces of the economy are the labour-productivity process for the worker household and the managerial-productivity process for the self-employed household. To identify quantitatively the effect of the change in the productivity process on the increasing wealth inequality, a dynamic general-equilibrium model of the educational, occupational, and consumption-saving choices is required. The model is that of an incomplete insurance market with four endogenous types of agents, two educational levels (non-college and college), and two occupational types (worker and self-employment). The model is an extension of that in Quadrini (2000), which studies the importance of entrepreneurship in accounting for wealth inequality in the United States. The major difference between the model in this paper and Quadrini's is that households in the economy invest in their direct descendant's education.

Each decision unit is a household with stochastic life. When a (parent) generation dies, the household is taken over by a new (child) generation. The educational level of the new generation is determined by the parent's investment decision in their child's education. As for the occupational choice, the household can choose to invest in a self-employment project in the next period. The self-employment project will carry a managerial-productivity shock that represents the uncertainty associated with being self-employed. Once the investment is realized in the following period, the household becomes self-employed and receives income from its business. The worker households earn wages by supplying their labour services, which are subject to labour-productivity shocks. The household can allocate its income among consumption, investment in the self-employment project, investment in the education of the next generation, and savings by lending its asset to the corporate (non-self-employment) production sector.

The fundamentals that drive this model economy are the labour-productivity process for worker households and the managerial-productivity process for self-employment households. These processes are assumed to be different between two educational levels. They will be calibrated to some earnings'

and one type of capital from the elasticity of substitution between skilled labour and the same type of capital. Their estimation results show that the elasticity of substitution between labour and capital is indeed lower for skilled labour than for unskilled labour. The lower elasticity of substitution implies that, as capital increases, there is a relatively higher demand for skilled labour than for unskilled labour, causing the skill premium to rise. This insight can be applied to the self-employment production technology. If the elasticity of substitution between the skilled (educated) manager and capital is lower than that of the unskilled (uneducated) and capital, then this could possibly account for the increase in wealth inequality driven by the college self-employed group. This way of accounting for the change in wealth inequality will not, however, be considered directly.

moments from the data, and will be the main focus of the calibration exercise in section 4. The model is described in detail in the following subsections.

3.1 Preferences

All households are assumed to be ex-ante homogeneous, which is to say that they have the same preferences. A household has a time-separable flow utility function with respect to its own consumption, represented by

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma},$$

where c_t represents the consumption at time t. Future utility is discounted by a factor, β.

3.2 Borrowing constraints

In their study of entrepreneurship and wealth distribution, Cagetti and De Nardi (2002) analyze the importance of borrowing constraints on the occupational distribution between workers and the self-employed, and on wealth concentration. In this paper, the households who invest in self-employment projects are assumed to face a borrowing constraint that depends on the household assets. Specifically, it is assumed that the household can invest at most $(1+\alpha)$, $\alpha \in [0,1)$, times its assets, a_t, in self-employment projects. Thus, $\alpha \cdot a_t$ is the amount of borrowing. The fact that $\alpha < 1$ implies that all the borrowings are collateralized by the household assets. All borrowing and lending are carried out at the common equilibrium interest rate of r_t.

3.3 Demographics

The households in this economy face a stochastic life. A generation of the household dies with probability ψ every period and thus survives with $(1-\psi)$. When the parent generation dies, a child generation is born and takes over the household. The child household inherits all the wealth from the previous generation. Since the occupational decision is made one period prior, if the parent household made the investment in the previous period to be self-employed in this period and the generation change were to occur in this period, it is assumed that the child household inherits the self-employment project with probability λ, and with probability $(1-\lambda)$ the child household starts out as a worker. Agents are assumed to be fully altruistic towards their offspring.

3.4 Education and occupation

The educational level is denoted by e. As briefly explained in section 3.3, the educational decision for a generation of the household is made by the parent generation. In modelling the educational investment decision, I follow Ríos-Rull and Sanchez-Marcos (2002), who compare several theories that can account for the gender difference in the college-attainment rate. A simplified version of their model of the educational investment decision is used in this paper. In any period, a parent household can invest i_e and the child generation of the household will be born with educational level e', given that the generational change occurs in the following period. The prime denotes the next-period value throughout the paper. If the parent generation does not die, in the next period the household faces the same decision of whether to invest i_e for the educational level, e', of the potential child generation next period. I denote this education production function as $h(i_e)$.

There are two occupations: worker and self-employed. Let k_t denote the amount of physical capital invested in a self-employment project. For the occupation, the worker is implied by $k_t = 0$ and self-employment by $k_t > 0$ in period t. Thus, $k_t = 0$ implies that there is no self-employment production for the household in period t. New generations of households start out their occupation as a worker, except in the case where the new generation successfully inherits the previous generation's self-employment project. The conditional probability at which this succession occurs is λ. In every period, a household decides on its next-period occupation by choosing the amount of self-employment investment, k', to make. When $k' = 0$, the household will become a worker in the following period; however, if $k' > 0$, it will become self-employed with the physical capital input, k', in the self-employment project.

When $k' = 0$, a worker, is chosen for the next period, the household inelastically supplies its labour endowment, ε_e, which is the efficiency units of labour, and the potential values it takes differ between two educational levels. The worker household receives labour earnings of $w \cdot \varepsilon_e$, where w is the wage rate per efficiency unit of labour. When $k' > 0$, the household becomes self-employed in the following period. Following Quadrini (2000), it is assumed that the self-employment production is subject to a productivity shock. The shock to the self-employment project occurs after the commitment of the household asset into the project. This set-up embodies the idea that running a self-employment project bears a risk. The self-employment productivity shock, denoted as η_e, is interpreted as managerial ability or productivity, as in Cagetti and De Nardi (2002). In this paper, however, the

values that η_e can take differ by the educational level, e, of the household. ε_e and η_e follow first-order Markov processes, $\Gamma_\varepsilon(\varepsilon'_e|\varepsilon_e)$ and $\Gamma_\eta(\eta'_e|\eta_e)$, respectively. When a change in occupation occurs during the life of one generation, the first-period value of ε_e or η_e for the new occupation is drawn from the respective unconditional distribution of the Markov processes. Similarly, when the generation change occurs, the first-period value of ε_e or η_e for the new generation is drawn from the respective unconditional distribution of the Markov processes.

3.5 Production technologies

There are two types of production technology in the economy: corporate production and self-employment production. The corporate production technology is represented by a Cobb-Douglas production function.

$$F(K, H) = K^{\gamma} N^{1-\gamma},$$

where K denotes the input of corporate sector physical capital and N denotes the sum of all workers' efficiency units of labour.

The self-employment production technology, on the other hand, is given by

$$f(k) = \eta k^{\nu},$$

where η is the managerial-productivity shock. As noted earlier, it follows a Markov process, $\Gamma_\eta(\eta'_e|\eta_e)$, where e is the educational level. A common depreciation rate of physical capital, δ, is assumed in both sectors.

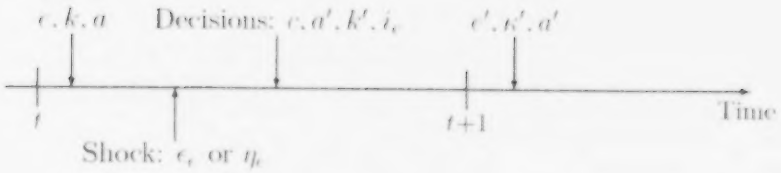
3.6 Return on self-employment project

Given $k > 0$, the household is identified as being self-employed. Once self-employed, the amount of the physical capital input is predetermined by the previous period's business investment decision. Given the realization of the productivity shock, η, the self-employment project profit is given by

$$\pi(\eta, k) = \eta k^{\nu} - (r + \delta) k, \tag{1}$$

where r is the risk-free rate of return on the investment in the corporate sector. rk is subtracted in equation (1) to explicitly show that the self-employment project capital is invested out of the

Figure 1: Timing of Events



household's asset, which has the opportunity cost rk. It can be easily observed that when the expected value of η is higher, the optimal amount of k' invested for the following period will be higher also, given that the variance of η is constant.

3.7 The household's problem

The household in this economy maximizes its expected lifetime utility. Figure 1 shows the timing of events. The horizontal line indicates the passage of time starting from period t. Households start the period with the given education (e), the self-employment project (k), and the household asset (a). A worker household ($k = 0$) observes ε_t, whereas a self-employed household ($k > 0$) observes η_t. Given these state variables $\{e, k, a, (\varepsilon_t \text{ or } \eta_t)\}$, the household makes the following decisions: consumption, c; total household savings, a'; self-employment project investment, k'; and educational investment, i_e.

In Figure 1, the self-employment project size for the $t + 1$ period is listed as κ' and not k', since the introduction of the stochastic life raises the issue of the child generation inheriting the self-employment project from the parent generation. In the $t + 1$ period, if the current generation survives with probability $1 - v$, then $e' = e$ and $\kappa' = k'$, implying that the education does not change and the occupation will be as decided in the previous period. If the old generation dies with probability v, the new generation of the household is born. In the first period after the new generation of the household is born, the household starts out as a worker if the previous generation of the household was a worker in its last period. If the previous generation was self-employed, then the

new generation of the household inherits the self-employment project with probability λ and becomes self-employed, or, with probability $1-\lambda$, the household becomes a worker and does not inherit the self-employment project, except for its invested net capital after depreciation. The educational level of the new generation of the household is decided according to the investment decision of the previous generation of the household, as specified above.

Since this paper focuses only on stationary equilibria where the distribution of households over the individual states are constant over time, the aggregate variables, such as the rate of return on capital and the wage rate for the efficiency units of labour, can be taken parametrically, as far as the household lifetime utility maximization is concerned. Those aggregate variables are, in turn, products of a stationary equilibrium. Hence, the relevant state variables for the household's optimization problem are ε, η, e, a, and k for the realization of the shock, the education, the household's total asset level, and the self-employment project, respectively.

I denote $v^{W,K}(\varepsilon,\eta,e,a)$ and $v^{SE}(\varepsilon,\eta,e,a,k)$ as the value function of the household for the worker and the self-employed, respectively. The household's lifetime utility maximization problem can then be written as follows.

Worker: (2)

$$v^{WK}(\varepsilon,\eta,e,a) = \max_{c,a',k',i_e} u(c)$$
$$+ \beta(1-v)\left[(1-I_{k'})\cdot Ev^{WK}(\varepsilon',\eta',e,a') + I_{k'}\cdot Ev^{SE}(\varepsilon',\eta',e,a',k')\right]$$
$$+ \beta v \left\{ \begin{array}{c} (1-I_{k'})\cdot Ev^{WK}(\varepsilon',\eta',e',a') \\ +I_{k'}\cdot\left[(1-\lambda)Ev^{WK}(\varepsilon',\eta',e',a') + \lambda Ev^{SE}(\varepsilon',\eta',e',a',k')\right] \end{array} \right\}$$

s.t. $c + a' + i_e = w\varepsilon + (1+r)a$,

$$e' = h(i_e),$$

$a' \geq 0$, and

$k' \leq (1+\alpha)a$, where

$$I_{k'} = \left\{ \begin{array}{ll} 0 & \text{if } k' = 0 \\ 1 & \text{if } k' > 0 \end{array} \right.$$

Self-employed: (3)

$$v^{SE}(\varepsilon,\eta,e,a,k) = \max_{c,a',k',i_e} u(c)$$
$$+\beta(1-e)\left[(1-I_{k'})\cdot Ev^{WK}(\varepsilon',\eta',e,a') + I_{k'}\cdot Ev^{SE}(\varepsilon',\eta',e,a',k')\right]$$
$$+\beta e\left\{\begin{array}{c}(1-I_{k'})\cdot Ev^{WK}(\varepsilon',\eta',e',a') \\ +I_{k'}\cdot\left[(1-\lambda)Ev^{WK}(\varepsilon',\eta',e',a') + \lambda Ev^{SE}(\varepsilon',\eta',e',a',k')\right]\end{array}\right\}$$

$$\text{s.t. } c + a' + i_e = \pi(\eta,k) + (1+r)a,$$
$$e' = h(i_e),$$
$$a' \geq 0, \text{ and}$$
$$k' \leq (1+\alpha)a, \text{ where}$$
$$I_{k'} = \begin{cases} 0 & \text{if } k' = 0 \\ 1 & \text{if } k' > 0 \end{cases}.$$

The conditions in maximization problems (2) and (3) are, in order, the budget constraint, the educational production function, the household total-asset borrowing constraint, and the self-employment project investment-liquidity constraint, respectively. r and w are the aggregate prices, the interest rate, and the wage rate, respectively. $\pi(\eta,k)$ is the self-employment project profit from equation (1). $I_{k'}$ is an indicator function, as defined at the end of problems (2) and (3). The expectation operator is taken with respect to $\Gamma_\varepsilon(\varepsilon'_e|\varepsilon_e)$ and $\Gamma_\eta(\eta'_e|\eta_e)$. As long as the occupation stays the same during a generation of the household, the productivity process will also stay $\Gamma_\varepsilon(\varepsilon'_e|\varepsilon_e)$ or $\Gamma_\eta(\eta'_e|\eta_e)$. When the occupation changes during the generation, the first value of the shock in the first period of the new occupation is drawn from the stationary distribution of the appropriate Markov process, $\Gamma_\varepsilon(\varepsilon'_e|\varepsilon_e)$ or $\Gamma_\eta(\eta'_e|\eta_e)$. I denote the stationary distributions of $\Gamma_\varepsilon(\varepsilon'_e|\varepsilon_e)$ and $\Gamma_\eta(\eta'_e|\eta_e)$ as $\Gamma^*_\varepsilon(\varepsilon_e)$ and $\Gamma^*_\eta(\eta_e)$, respectively. As noted earlier, in solving (2) and (3), the households take r and w as given. Section 4 provides the precise definition of the stationary equilibrium of this economy. The definition of the equilibrium is given in Appendix C. For the quantitative analysis that follows, the computational method used to calculate the equilibrium is also described in Appendix C.

4 Calibration

I use the following general calibration strategy. The model parameters are calibrated to the selected aggregate moments from the 1983 SCF. The distribution of household types, and the earnings and wealth moments, are the focus of the calibration. After calibrating to the 1983 economy, a subset of the model parameters, which dictates the productivity processes, is recalibrated to emulate the economy in terms of the household type distribution and the earnings moments from the 2001 SCF. The results of the 1983 model economy and the 2001 model economy are compared in terms of the differences in between-group wealth inequality. This analysis reveals how much of the change in wealth inequality over 1983 2001 can be accounted for by the change in the household type distribution and the change in earnings inequality.

First, a baseline model is calibrated to the 1983 data, so that the model economy can replicate the statistical moments obtained from the data in several important dimensions. A subset of parameters and selected moments are, by their nature, independent of the values of other parameters and moments. I call them independent parameters. The rest of the parameters are interdependent in influencing the selected moments that come out in the equilibrium. These parameters will be calibrated together with the calculation of the stationary equilibrium. I call them dependent parameters.

I make several assumptions to carry out this quantitative exercise. The state variables $(\varepsilon, \eta, e, a, k)$ and the choice variables (i_e, a', k') are discretized. The education levels, e, are discretized to two, $e \in \{\text{non-college,college}\}$. The number of finite elements, ε or η, is assumed to be two for each educational level. Thus, the first-order Markov processes for ε and η are 2-by-2 matrices. These processes are assumed to be identical between the two educational levels. The household assets, a and a', are discretized over 1,000 grid points. The self-employment project sizes are discretized to four values, $k \in \{0, k_1, k_2, k_3\}$. The educational investment choice, i_e, is assumed to take two discrete values, $i_e \in \{0, i_{co}\}$, where i_{co} denotes the amount of investment for college education. In addition, the education production function is assumed to take the following form:

$$h(i_e) = \begin{cases} \text{college} & \text{if } i_e = i_{co}, \\ \text{non-college} & \text{if } i_e = 0. \end{cases}$$

The flow utility takes the constant relative risk-aversion (CRRA) form, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The model period is assumed to be a year. The parameters to be calibrated are listed in Table 13. The

Table 13: List of Parameters

Description	Parameter
Demography	ω
Preferences	β, σ
Liquidity	α
Labour productivity	$\left\{\varepsilon^1_{nc}, \varepsilon^2_{nc}, \varepsilon^1_{co}, \varepsilon^2_{co}\right\}, \Gamma_\varepsilon$
Managerial productivity	$\left\{\eta^1_{nc}, \eta^2_{nc}, \eta^1_{co}, \eta^2_{co}\right\}, \Gamma_\eta$
SE production	$\{k_1, k_2, k_3\}, \nu, \lambda_1$
Corporate production	γ, δ
Education	i_{nc}, i_{co}, Γ_e

Table 14: Independent Parameters

Parameter	Value	Source
ω	$\frac{1}{35}$	35 yrs. ave. work-life
α	0.5	Evans and Jovanovic 1989
λ	0.28	Dunn and Holtz-Eakin 2000
$\frac{k_2}{k_1}\left(=\frac{k_3}{k_2}\right)$	10	Quadrini 2000

subscripts nc and co denote non-college and college households, respectively. Some of the parameters are assigned a value standard in the literature. I assign $\sigma = 2$, $\gamma = 0.33$, and $\delta = 0.062$, for the risk-aversion parameter, the capital share of the income in the corporate sector, and the depreciation rate of physical capital, respectively. The lowest value of labour productivity for non-college households is normalized to one, $\varepsilon^1_{nc} = 1$. Table 14 lists the independent parameters with the assigned values: ω is assigned $\frac{1}{35}$, which gives a generation of the household 35 years of working life, on average. Evans and Jovanovic (1989) study the significance of the borrowing constraint in the household's decision to become self-employed. Their results show that the borrowing constraint significantly affects occupational choice; they estimate that the maximum amount a household can borrow to invest in a self-employment project is about 50 per cent of the total household assets. Based on their results, α is set at 0.5. The probability of successfully inheriting a self-employment project between generations is set at 0.28, which, in Dunn and Holtz-Eakin's (2000) study of the National Longitudinal Surveys of Labour Market Experience, is the self-employment rate of a son given that the father was self-employed. The ratio of the self-employment project sizes is assumed to be constant between $\frac{k_2}{k_1}$ and $\frac{k_3}{k_2}$, and equals 10 following Quadrini (2000). Thus, by determining one of the self-employment

Table 15: Dependent Parameters

Parameter	Value
β	0.904
$\{\varepsilon^1_{nc}, \varepsilon^2_{nc}, \varepsilon^1_{co}, \varepsilon^2_{co}\}$	$\{1.000, 3.671, 1.001, 6.450\}$
$\Gamma_\varepsilon(\varepsilon'\|\varepsilon)$	$\begin{bmatrix} 0.614 & 0.386 \\ 0.395 & 0.605 \end{bmatrix}$
$\{\eta^1_{nc}, \eta^2_{nc}, \eta^1_{co}, \eta^2_{co}\}$	$\{0.270, 1.326, 0.300, 1.473\}$
$\Gamma_\eta(\eta'\|\eta())$	$\begin{bmatrix} 0.935 & 0.065 \\ 0.213 & 0.787 \end{bmatrix}$
k_3	159.853
ν	0.708
i_{co}	0.290

project sizes, all three are determined. The value of k_3 is calibrated below.

The rest of the parameters are calibrated by targeting the moments from the model equilibrium to match as closely as possible those of the data. In judging the calibration results, it is important to keep in mind that the model being analyzed is highly non-linear, such that a perfect match of all moments is very unlikely. The calibration results reported are from the closest match obtained given this consideration.

Table 15 shows these parameters and the results of the calibration. Since each row of Γ_ε and Γ_η has to sum to one, there are only two values to pin down for each Γ_ε and Γ_η. In total, there are 15 free parameters left to be calibrated. The 15 moments used to pin down these parameters are the type distribution of the households (four types, implying three moments), the relative average earnings (three moments), the coefficients of variation of earnings by type (four moments), the relative average wealth (three moments), the fraction of self-employed income, and the interest rate. The values of these moments are calculated using the 1983 SCF, except for the interest rate, and reported in Table 16. The interest rate target is set at 4 per cent per year, which is a standard value used in the literature as an average return on bonds, and as the return on equity in the United States.

The results of the calibration show that many of the moments are well captured, especially the household type distribution. Some moments, however, are difficult to match. For the relative average earnings, three groups' values are close to the target, except for the non-college self-employed household, which is too high in the model at 0.788 compared with the data at 0.48. For the coefficient of variation, the value for the college self-employed household is close to the target, but other moments

Table 16: Calibration Results of the Baseline Model

Moment		Model	Data
Type distribution:	Non-college worker	0.61	0.61
	Non-college self-employed	0.10	0.09
	College worker	0.25	0.26
	College self-employed	0.05	0.04
Relative ave. earnings:	Non-college worker	0.50	0.52
	Non-college self-employed	0.79	0.48
	College worker	0.80	0.86
	College self-employed	1.00	1.00
CV of earnings:	Non-college worker	0.57	0.68
	Non-college self-employed	0.85	0.77
	College worker	0.72	0.94
	College self-employed	1.01	1.03
Relative ave. wealth:	Non-college worker	0.17	0.27
	Non-college self-employed	0.54	0.61
	College worker	0.42	0.52
	College self-employed	1.00	1.00
SE income fraction		0.30	0.16
Interest rate		0.07	0.04

do not match very closely. For the relative average wealth, the order of the four groups' average wealth is captured, even though the values are off target by small amounts. The fraction of self-employed income over all income is higher in the model at 0.30 relative to the data at 0.16. The interest rate is high in the model economy at 7 per cent relative to 4 per cent in the model.

Even though some of the target moments are not captured in the calibration exercise, my main goal in this paper is to measure the changes in wealth inequality and not wealth inequality itself. Thus, I take these calibration results to represent the dimensions of interest for the baseline economy in 1983. In section 5, I recalibrate the productivity processes for 2001 to measure the changes in wealth inequality that result from the changes in earnings inequality.

5 Quantitative Results

In this section, I recalibrate a subset of the parameters that dictate the labour and managerial-productivity processes, to match the earnings moments from the 2001 SCF data. Specifically, in this part of the calibration, I do not target any wealth-related moments. I compare the results for between-group wealth inequality from this calibration with those of the 1983 calibration, and then

Table 17: Recalibrated parameters, 2001

Parameter	Value
$\{\varepsilon^1_{nc}, \varepsilon^2_{nc}, \varepsilon^1_{co}, \varepsilon^2_{co}\}$	$\{1.000, 3.597, 1.037, 6.568\}$
$\Gamma_\varepsilon(\varepsilon'\|\varepsilon)$	$\begin{bmatrix} 0.588 & 0.412 \\ 0.396 & 0.604 \end{bmatrix}$
$\{\eta^1_{nc}, \eta^2_{nc}, \eta^1_{co}, \eta^2_{co}\}$	$\{0.075, 1.663, 0.273, 1.597\}$
$\Gamma_\eta(\eta'\|\eta)$	$\begin{bmatrix} 0.935 & 0.065 \\ 0.206 & 0.794 \end{bmatrix}$
i_{co}	0.309

Table 18: Calibration Results, 2001

Moment		Model	Data
Type distribution:	Non-college worker	0.60	0.56
	Non-college self-employed	0.05	0.09
	College worker	0.29	0.28
	College self-employed	0.06	0.07
Relative ave. earnings:	Non-college worker	0.37	0.26
	Non-college self-employed	0.73	0.38
	College worker	0.62	0.55
	College self-employed	1.00	1.00
CV of earnings:	Non-college worker	0.56	0.73
	Non-college self-employed	1.42	2.77
	College worker	0.71	1.84
	College self-employed	1.37	2.03
Self-employed income fraction		0.36	0.27
Interest rate		0.06	0.04

analyze how much of the change in wealth inequality over 1983–2001 is accounted for by the change in the labour and managerial-productivity processes, which directly affect earnings inequality.

Table 17 shows the subset of the parameters that are recalibrated and their calibrated values. Again, $\varepsilon^1_{nc} = 1$ is assigned as the normalization. There are 12 parameters for this stage of calibration. Table 18 shows the 12 moments used to match the model outcome and the data. As is the case for the 1983 calibration, the type distribution and relative earnings match fairly well, except for non-college self-employed households. It is also difficult to match the coefficients of variation of earnings. Table 19 shows the relative average wealth from the model and the data. These are the moments that are not the targets for the 2001 calibration. The table shows that the model-produced relative average

Table 19: Relative Average Wealth, 2001

Moment		Model	Data
Relative ave. wealth:	Non-college worker	0.09	0.06
	Non-college self-employed	0.54	0.27
	College worker	0.29	0.22
	College self-employed	1.00	1.00

Table 20: Percentage Change in Average Wealth, 1983–2001, Data

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	-43	8	99
College	4	146	137
College/Non-college	90	126	

Table 21: Percentage Change in Average Wealth, 1983–2001, Model

Edu/occ	Worker	Self-employed	Self-employed worker
Non-college	-19	55	92
College	6	56	47
College/Non-college	31	0.4	

wealth of non-college self-employed households is particularly different from the data: the coefficient of variation of earnings for this group is very high in 2001 relative to that of the other groups. Since the calibration exercise tries to match the coefficient of variation moments, the agents' increased precautionary motive leads to a higher savings rate and thus higher wealth. This is the key insight that drives the following results.

Based on the calibration results, I can compare the percentage changes in the average wealth between 1983 and 2001. Table 20 is a reproduction of Table 12 showing the percentage change in the average wealth observed in the data; Table 21 shows figures obtained from the model. Comparing Table 21 with Table 20, the percentage change in average wealth for the non-college worker is -19 per cent in the model, as opposed to -43 per cent in the data. Thus, the absolute value of the change is much smaller in the model for this group. For the non-college self-employed, it is 55 per cent in the model relative to 8 per cent in the data. The model-produced change in average wealth is much larger than is observed in the data. As stated earlier, this model result is driven by the increase in the coefficient of variation in earnings for this group. For the college worker, the values are 6 per

cent and 4 per cent for the model and the data, respectively. For the college self-employed, it is 56 per cent in the model compared with 146 per cent in the data. Hence, the model-implied increase in the average wealth for this group is much smaller than is observed in the data.

In terms of relative wealth conditional on education, the relative average wealth of non-college self-employed to non-college worker households increases by 92 per cent in the model, and by 99 per cent in the data. In this dimension, the model captures a big part of the relative wealth change. Thus, between these two groups, the increase in wealth inequality derives mostly from the change in the earnings opportunity that gives rise to the observed change in the household type distribution and the earnings inequality. Between the college self-employed and the college worker, wealth inequality increases by 47 per cent in the model and by 137 per cent in the data. Thus, the increased earnings opportunity for college self-employed households accounts for one-third of the change in the relative average wealth between college self-employed and college workers.

Conditional on occupation, the relative average wealth of the college worker to the non-college worker has increased by 31 per cent in the model, as opposed to 90 per cent in the data; thus, the model captures about one-third of the observed change. On the other hand, the relative average wealth of the college self-employed to non-college self-employed household has increased by 0.4 per cent in the model, compared with 126 per cent in the data. Thus, the increased earnings opportunity for college self-employed does not lead to the observed change in the relative average wealth between college self-employed and non-college self-employed households.

6 Discussion

The results described in section 5 show that some of the observed changes in wealth inequality among the household groups considered did not derive from the observed changes in the earnings inequality among them, especially in the area of wealth inequality between college self-employed and non-college self-employed households. In the following subsections, some potential extensions of the model are discussed, to set a path to further investigate the likely sources of the changes in wealth inequality. As potential sources, two alternative institutional changes occurred during the time period: the U.S. Tax Reform Act of 1986 and the increase in small-business loan guarantees by the government. Each of these sources is qualitatively discussed as a possible reason for the observed increase in wealth inequality.

6.1 Income tax

The Tax Reform Act of 1986 made the marginal tax rate less progressive. This type of tax change can be expected to induce the earnings-rich college self-employed households to save more. The distributional implications of tax policy changes have been analyzed in the literature. Meh (2002) investigates the wealth distributional effects of a tax policy change from a progressive to a proportional income tax system when entrepreneurship is considered.[7] His main finding is that the tax reform leads to an increase in wealth inequality in an economy without entrepreneurs, but that this increase will be much smaller when entrepreneurial activities are added to the model. Even though this result casts some doubt on whether the Tax Reform Act of 1986 made wealth inequality more dispersed in the economy with entrepreneurs, Meh (2002) explains that the result is based on the fact that, when the tax policy changes, there is a reduction in the between-group wealth inequality of entrepreneurs and non-entrepreneurs through a general-equilibrium effect. Thus, wealth inequality among entrepreneurs (i.e., college entrepreneurs and non-college entrepreneurs) would still increase.

6.2 Small-business loans

A complementary explanation would be an increase in small business loans. Li (2002) analyzes how current and alternative government credit subsidy policies affect entrepreneurial decisions. She finds that these policies can have large allocational effects along with effects on occupational choices. Empirically, the size of the government loan guarantees are large. For example, the U.S. Small Business Administration provides small-business loan guarantees. According to their annual reports for 1983 and 2001, the gross value of the loan guarantees in 1983 was $6.7 billion, and in 2001 it was $46.9 billion.[8] This is a seven-fold increase in the amount of loans over this period. Since the loans policy is directed towards smaller businesses, it would especially relax the liquidity constraints of the self-employed who run smaller businesses. Non-college self-employed households who, on average, run smaller businesses would benefit relatively more from this increase. As liquidity constraints become less severe, the incentive for savings declines. This helps to account for the increasing gap in average wealth between college self-employed and non-college self-employed households.

[7] Cagetti and De Nardi (2004) also examine the distributional effects of a tax policy change. They study the effects on wealth distribution of a change in both the estate tax and the income tax. They conclude that abolishing the estate tax and increasing the income tax to balance the government budget constraint would redistribute wealth to the richest from other households.

[8] I approximate the value of the loan guarantees for 1983 using the information contained in the 1983 annual report.

The extensions discussed in sections 6.1 and 6.2 should help to account for the observed changes in wealth inequality over the period. Further quantitative investigation into this area is required, however, to determine how much of the change is accounted for.

7 Conclusions

Over the 1983–2001 period, there was a big increase in the earnings of college self-employed households over other groups, and a much bigger concentration of their wealth. In this paper I have investigated whether the concentration of wealth in college self-employed households is a result of the higher relative earnings for this group.

A general-equilibrium model of wealth distribution with education and occupation choices was constructed, where four household groups were considered: college self-employed, college worker, non-college self-employed, and non-college worker. The model was calibrated to match the household type distribution, the relative earnings, and the relative wealth in 1983. A subset of the model parameters was recalibrated to match the earnings observations in 2001. In the 2001 calibration, the wealth observations were not targeted intentionally, since the result of this exercise would identify the changes in wealth inequality that specifically came from the changes in earnings inequality. The changes in relative average earnings were found to account for one-third of the changes in relative average wealth between college self-employed and college worker households; however, almost none of the changes in the relative average wealth between college self-employed and non-college self-employed households were accounted for. The change in wealth inequality between college self-employed and non-college self-employed households was not captured in the model because the variation of earnings for non-college self-employed increased dramatically in 2001 relative to that for college self-employed. Since the calibration exercise tries to match the coefficient of variation in earnings moment, the agents' increased precautionary motive leads to a higher savings rate and thus higher wealth.

I have identified two institutional changes that could further account for the changes in observed wealth inequality. The first is the change in the tax system (i.e., the Tax Reform Act of 1986), which made business taxes less progressive. This change allowed the college self-employed households to increase their savings over other groups; they are the highest earnings group of all. The second institutional change is the increase in small-business loan guarantees by the government. This reduction in the liquidity constraints towards smaller businesses would influence disproportionately the

non-college self-employed households to reduce their precautionary savings. Qualitatively speaking, these two institutional changes seem to hold promise in accounting for the relative average wealth changes observed over this period. Still, the quantitative results would have to be investigated in the future before a precise statement could be made regarding the effects of institutional changes on wealth inequality.

References

Aiyagari, S.R. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics* 109: 659–84.

Aizcorbe, A., A.B. Kennickell, and K.B. Moore. 2003. "Recent Changes in U.S. Family Finances: Evidence from the 1998 and 2001 Survey of Consumer Finances." *Federal Reserve Bulletin*, January.

Bound, J. and G. Johnson. 1992. "Changes in the Structure of Wages in the 1980's: An Evaluation of Alternative Explanations." *American Economic Review* 82: 371–92.

Budría-Rodriguez, S., J. Díaz-Giménez, V. Quadrini, and J.V. Ríos-Rull. 2002. "Updated Facts on the U.S. Distributions of Earnings, Income, and Wealth." Federal Reserve Bank of Minneapolis *Quarterly Review* (Summer) 26(3): 2–35.

Cagetti, M. and M. De Nardi. 2002. "Entrepreneurship, Frictions and Wealth." Federal Reserve Bank of Minneapolis Working Paper No. 620.

—. 2004. *Taxes, Entrepreneurship, and Wealth*. Federal Reserve Bank of Minneapolis Staff Report No. 340.

Castañeda, A., J. Díaz-Giménez, and J.V. Ríos-Rull. 2003. "Accounting for the U.S. Earnings and Wealth Inequality." *Journal of Political Economy* 111: 818–57.

Díaz-Giménez, J., V. Quadrini, and J.V. Ríos-Rull. 1997. "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth." Federal Reserve Bank of Minneapolis *Quarterly Review* (Spring) 21: 3–21.

Dunn, T. and D. Holtz-Eakin. 2000. "Financial Capital, Human Capital, and the Transition

to Self-Employment: Evidence from Intergenerational Links." *Journal of Labor Economics* 18: 282–305.

Evans, D.S. and B. Jovanovic. 1989. "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints." *Journal of Political Economy* 97: 808–27.

Gentry, W.M. and G.R. Hubbard. 2000. "Entrepreneurship and Household Saving." National Bureau of Economic Research Working Paper No. 7894.

Gottschalk, P. and R. Moffitt. 1994. "The Growth of Earnings Instability in the U.S. Labor Market." *Brookings Papers on Economic Activity* 2: 217–72.

Holtz-Eakin, D., D. Joulfaian, and H.S. Rosen. 1994. "Entrepreneurial Decision and Liquidity Constraints." *Rand Journal of Economics* 25(2): 334–47.

Huggett, M. 1993. "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies." *Journal of Economic Dynamics and Control* 17: 953-69

Hurst, E. and A. Lusardi. 2004. "Liquidity Constraints, Household Wealth, and Entrepreneurship." *Journal of Political Economy* 112: 319–347.

Katz, L.F. and K.M. Murphy. 1992. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *Quarterly Journal of Economics* 107: 35–78.

Kennickell, A.B. and M. Starr-McCluer. 1994. "Changes in Family Finances from 1989 to 1992: Evidence from the Survey of Consumer Finances." *Federal Reserve Bulletin* 80: 861–82.

Krueger, D. and F. Perri. 2005. "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory." *Review of Economic Studies* 73: 163–193.

Krusell, P., L.E. Ohanian, J.V. Ríos-Rull, and G.L. Violante. 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica* 68: 1029–53.

Li, W. 2002. "Entrepreneurship and Government Subsidies: A General Equilibrium Analysis." *Journal of Economic Dynamics and Control* 26: 1815–44.

Meh, C.A. 2002. "Entrepreneurship, Inequality, and Taxation." Bank of Canada Working Paper No. 2002–14.

Moore, K. 2003. "Comparing the Earnings of Employees and the Self-Employed." Unpublished manuscript. Board of Governors of the Federal Reserve System.

Pence, M.K. 2001. "401(k)s and Household Saving: New Evidence from the Survey of Consumer Finances." Federal Reserve Board of Governors, Finance and Economics Discussion Paper No. 2002–6.

Quadrini, V. 1999. "The Importance of Entrepreneurship for Wealth Concentration and Mobility." *Review of Income and Wealth* 45(1): 1–19.

———. 2000. "Entrepreneurship, Saving, and Social Mobility." *Review of Economic Dynamics* 3: 1–40.

Ríos-Rull, J.-V. and V. Sanchez-Marcos. 2002. "College Attainment of Women." *Review of Economic Dynamic* 5(4): 965–80.

U.S. Small Business Administration. 1983. *Annual Report.*

———. 2001. *Performance & Accountability Report.*

Wolff, E.N. 1995. *Top Heavy: A Study of the Increasing Inequality of Wealth in America*. New York: Twentieth Century Fund Press.

—. 1996. "International Comparisons of Wealth Inequality." *Review of Income and Wealth* 42(4): 433–51.

Appendix A: The SCF Data

The Survey of Consumer Finances is conducted triennially by the Federal Reserve Board. It collects information on representative U.S. households regarding demographics, earnings, income, and wealth. The years used in this paper are 1983 and 2001. The sample sizes are 3,143 and 4,442 households, respectively, for 1983 and 2001. The SCF uses a two-part sample design: an area probability sample and a "list" sample. The area probability sample follows the U.S. Census Bureau's national sampling frame. The list sample uses the IRS Statistics of Income Individual Taxpayer File to sample wealthy households in the population. Partly as a result of this sampling design, the SCF provides wealth data that are widely regarded as the most comprehensive data available for the United States.

The subsample used in this paper restricts the head of the household to be between the age of 25 and 65, working 20 hours or more a week at the time of the survey, and have job tenure of one or more years. The last restriction is placed to keep consistency between earnings and occupation: there is a time lag between the information on earnings and the "current" main job. While the current main job is that at the time of the survey, the annual earnings data are from the previous year. The resulting sample sizes vary from 2,314 in 1983 to 2,706 in 2001.

Appendix B: Definitions

College: A college household is one whose head has a college degree level of education or higher. Hence, a non-college household is defined to be one without a college degree.

Earnings[9]: The definition of earnings follows Budría-Rodríguez et al. (2002):

> We define labour earnings as wages and salaries of all kinds plus a fraction of business income. Business income includes income from professional practices, businesses, and farm sources. The value for the fraction of business and farm income that we impute to labour earnings is the samplewide ratio of unambiguous labour income (wages plus salaries) to the sum of unambiguous labour income and unambiguous capital income. This ratio is 0.857 for the 1998 SCF sample. (For the 1992 SCF sample, this ratio was 0.864.)

Self-employed: Self-employment exists when the main job of the head of a household is self-reported as self-employment. Several different definitions of the term are used in the literature. For example, Quadrini (2000) defines self-employed to mean households who own a business or have a financial interest in some business enterprise. Cagetti and De Nardi (2002) restrict self-employed households to those who declare owning a business and having an active management role in it. Gentry and Hubbard (2000) restrict the term self-employed further by selecting households who own at least $5,000 in actively managed business. In this paper, since a self-employed household in the model is characterized by those who face a different stochastic process (i.e., managerial productivity shocks), the self-report of the main job is used to select the households who face these self-employment earnings risks, regardless of whether the household declares owning a business. A household whose head is currently working but not self-employed is thus defined to be a worker.

Wealth[10]: The definition of wealth follows that used by Budría-Rodríguez et al. (2002):

[9]Moore (2003) provides an alternative calculation of the earnings of self-employed by directly using the retained business earnings in the SCFs.

[10]This definition of wealth does not include defined-benefit pension plans. Pence (2001) imputes the present value of these plans using the SCFs. Including them does not change the general trend given by the education-occupation specific average wealth.

We define wealth as the net worth of the households. Our definition includes the value of financial and real assets of all kinds net of various kinds of debts. Specifically, the assets that we consider are the following: residences and other real estate; farms and all other businesses; checking accounts, certificates of deposit, and other banking accounts; IRA/Keogh accounts, money market accounts, mutual funds, bonds and stocks, cash and call money at the stock brokerage, and all annuities, trusts and managed investment accounts; vehicles; the cash value of term life insurance policies and other policies; pension plans accumulated in accounts; and other assets. The debts we consider are housing debts, such as mortgages and home equity loans and lines of credit; other residential property debts, such as those derived from land contracts and vacation residences; credit card debts; installment loans; loans taken against pensions; loans taken against life insurance; margin loans; and other miscellaneous debts. Our definition of wealth differs slightly from those used in other studies. Wolff (1995), for instance, provides several definitions of household wealth. Wolff's (1995) definition that is closest to ours is what he calls marketable wealth. The main difference between this definition and ours is that Wolff does not include vehicles and pension plans accumulated in accounts, and we do. Kennickell and Starr-McCluer's (1994) definition differs from ours in that they include the current face value of term life insurance policies that build up a cash value (that is, the cash amount paid in case the insured event occurs), while ours includes only the cash value of these policies.

Appendix C: Definition of Equilibrium

Define Ω to be the set of all state variables, ε, η, e, a, and k. Denote by $\mu(\varepsilon, \eta, e, a, k)$ the measure of households over the state variables. Let B be an appropriate family of subsets of Ω.

Definition 1 *A stationary recursive competitive equilibrium for this economy is the following list of objects:*

- *Value functions:* $v^{WK}(\varepsilon, e, a)$ *and* $v^{SE}(\eta, e, k, a)$;
- *Decision rules:*

$$c^{WK}(\varepsilon, \eta, e, a), \quad a'^{WK}(\varepsilon, \eta, e, a), \quad k'^{WK}(\varepsilon, \eta, e, a), \quad i_e^{WK}(\varepsilon, \eta, e, a),$$
$$c^{SE}(\varepsilon, \eta, e, a, k), \quad a'^{SE}(\varepsilon, \eta, e, a, k), \quad k'^{SE}(\varepsilon, \eta, e, a, k), \text{ and } i_e^{SE}(\varepsilon, \eta, e, a, k);$$

- *Rates of return:* $\pi(\eta, k)$, r, *and* w;
- *Capital and labour demands from the corporate production sector:* K *and* H; *and*
- *A transition function,* $\Psi(.)$, *which maps the space of* μ *into the same space,* $\mu' = \Psi(\mu)$;

such that the following conditions hold:

- $v^{WK}(.)$ *and* $v^{WK}(.)$ *are defined in (2) and (3);*
- $c^{WK}(.)$, $a'^{WK}(.)$, $k'^{WK}(.)$, *and* $i_e^{WK}(.)$ *solve the household problem (2), and* $c^{SE}(.)$, $a'^{SE}(.)$, $k'^{SE}(.)$, *and* $i_e^{SE}(.)$ *solve the household problem (3);*
- r *and* w *are determined competitively in the aggregate production sector, such that*

$$\begin{aligned} r &= F_1(K, N) - \delta, \text{ and} \\ w &= F_2(K, N), \text{ respectively.} \end{aligned}$$

and $\pi(\eta, k)$ *is determined by (1);*

- *Markets clear: there are two markets, (i) the capital market, and (ii) the labour market:*

(i)

$$K + \int_{\varepsilon,\eta,e,k,a} k\mu(\varepsilon, \eta, e, da, k) = \sum_{\varepsilon,\eta,e,k} \int_a a \, d\mu;$$

(ii)

$$N = \int_{\varepsilon,\eta,e,a} \varepsilon \, d\mu(\varepsilon, \eta, e, a, 0);$$

- *The stationary distribution of the households, μ, is the fixed point of the law of motion, $\mu' = \Psi(\mu)$. The transition function is constructed consistently with the individual household's decision rules and all the exogenous stochastic processes that, for all $(B_\varepsilon \times B_\eta \times B_e \times B_k \times B_a) \in B$, such that*

$$\mu'(B_\varepsilon \times B_\eta \times B_e \times B_k \times B_a) = \Psi(B_\varepsilon \times B_\eta \times B_e \times B_k \times B_a)(\mu) =$$

$$\int_{\substack{\varepsilon' \in B_\varepsilon, \\ \eta' \in B_\eta, \\ e' \in B_e, \\ k' \in B_k \\ a' \in B_a}} \left\{ \begin{array}{c} \int_{\varepsilon,\eta,e,k,a} \left[(1-e)\ I(\varepsilon,\eta,e,a,k)\ \Gamma^0_{\varepsilon,\eta}(\varepsilon',\eta'|\varepsilon,\eta,k',k) \right] d\mu \\ + \\ \int_{\varepsilon,\eta,e,k,a} e \left[\begin{array}{c} \lambda\, I(\varepsilon,\eta,e,a,k)\ \Gamma^1_{\varepsilon,\eta}(\varepsilon',\eta'|k') + \\ (1-\lambda)\ I_0(\varepsilon,\eta,e,a,k)\ \Gamma^s_\varepsilon(\varepsilon') \end{array} \right] d\mu \end{array} \right\} d\varepsilon'\, d\eta'\, de'\, da'\, dk',$$

where $I(\varepsilon,\eta,e,a,k)$ and $I_0(\varepsilon,\eta,e,a,k)$ are indicator functions defined to be

$$I(\varepsilon,\eta,e,k,a) = \left\{ \begin{array}{ll} 1 & \text{if } \left[\begin{array}{c} h[i_e(\varepsilon,\eta,e,a,k)] \in B_e, \\ k'(\varepsilon,\eta,e,a,k) \in B_k \text{ and} \\ a'(\varepsilon,\eta,e,a,k) \in B_a \end{array} \right] \\ 0 & \text{otherwise} \end{array} \right.$$

$$I_0(\varepsilon,\eta,e,k,a) = \left\{ \begin{array}{ll} 1 & \text{if } \left[\begin{array}{c} h[i_e(\varepsilon,\eta,e,a,k)] \in B_e, \\ a'(\varepsilon,\eta,e,a,k) \in B_a \end{array} \right] \text{ and } k' = 0 \\ 0 & \text{otherwise} \end{array} \right.$$

Also, $\Gamma^0_{\varepsilon,\eta}$ and $\Gamma^1_{\varepsilon,\eta}$ are defined as, respectively.

$$\Gamma^0_{\varepsilon,\eta}(\varepsilon',\eta'|\varepsilon,\eta,k',k) = \left\{ \begin{array}{ll} \Gamma_\varepsilon(\varepsilon'|\varepsilon) & \text{if } k' = 0 \text{ and } k = 0 \\ \Gamma_\eta(\eta'|\eta) & \text{if } k' > 0 \text{ and } k > 0 \\ \Gamma^s_\varepsilon(\varepsilon') & \text{if } k' = 0 \text{ and } k > 0 \\ \Gamma^s_\eta(\eta') & \text{if } k' > 0 \text{ and } k = 0 \end{array} \right. \text{ and}$$

$$\Gamma^1_{\varepsilon,\eta}(\varepsilon',\eta'|k') = \left\{ \begin{array}{ll} \Gamma^s_\varepsilon(\varepsilon') & \text{if } k' = 0 \\ \Gamma^s_\eta(\eta') & \text{if } k' > 0 \end{array} \right. .$$

Appendix D: Computational Method

In calculating the stationary equilibrium of the model built in this paper, the state-space discretization method is used to approximate the continuous household asset state space. One thousand discrete grid points for asset holdings are assigned from the lower bound of zero (no borrowing) to the upper bound that is determined so that the household savings decision at the upper bound is to save less than what it started the period with. The distance between consecutive grid points is set to be finer at lower levels of assets and coarser at higher levels. Given this discretization of the asset state space, the household's dynamic programming problem is such that the value function is defined over 16,000 finite state points.

Given a set of all parameter values, the stationary equilibrium of the model is calculated as follows. First, guess the equilibrium interest rate, r_0. From the corporate sector production function, the capital-labour ratio, as well as the wage rate, w, are determined. The household problem is then solved taking as given the prices of the economy, r and w. Using the decision rules from the household problem, the transition function of the distribution of households over the state space is constructed. The stationary distribution of the households, $\tilde{\mu}$ such that $\tilde{\mu} = \Psi(\tilde{\mu})$, is calculated by iterating the probability measure μ. Once the fixed point of this process is obtained, the new interest rate, r_1, is calculated from $\tilde{\mu}$. If the difference between r_0 and r_1 is greater than the specified threshold value, the procedure is repeated using r_1 as the new guess until the interest rate converges.

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